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CURVES IN COMPUTER GRAPHICS

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Abstract: *This paper explain a definition of different curves that are used in computer graphics. Mathematic describe of different curves and their main characteristic will be describe .*

Key words: *Curves, Surfaces.*

1. INTRODUCTION

There are many types of curves used in computer graphics. In computer graphics, curves and surfaces are fundamental concepts used to represent and render objects in a virtual environment. Curves are one-dimensional representations that are defined by a set of control points and mathematical equations. They are commonly used to represent smooth and continuous shapes, such as lines, arcs, and splines. Curves can be rendered using various algorithms, such as Bézier curves or B-splines, which interpolate the control points to create a smooth curve.

On the other hand, surfaces are two-dimensional representations that define the shape and geometry of objects in computer graphics. They are typically composed of a network of curves or a set of control points that define the surface's shape. Surfaces can be used to represent complex objects like 3D models, terrain, or character meshes. Rendering surfaces involves techniques like polygonal mesh rendering, where the surface is approximated by a collection of polygons, or more advanced methods like ray tracing or physically-based rendering.

The main difference between curves and surfaces lies in their dimensionality. Curves exist in one dimension and are used to represent linear or curved shapes, while surfaces exist in two dimensions and are used to represent more complex and volumetric objects. Additionally, the rendering techniques for curves and surfaces may differ, as curves can be rendered using interpolation algorithms, while surfaces often require more complex rendering algorithms to accurately represent their geometry and appearance. It's important to note that both curves and surfaces play crucial roles in computer graphics, and their rendering techniques continue to evolve with advancements in technology and algorithms. A curve is an infinitely large set of points. Each point has two neighbors except endpoints. Curves can be broadly classified into several categories:

- implicit,
- explicit,
- parametric curves,
- Bezier curves,
- B-spline curves.

2. IMPLICIT CURVES

An implicit curve or surface is the set of zeros of a function of 2 or 3 variables. Implicit curve functions are used to define lines and planes. Provides no control over tangents at connection points when joining several implicit functions. Implicit functions are hard to find for many shapes. Use a function that states which points are on and off the curves. The definition of implicit curves:

- All lines are defined with: Ax+By+C=0,
- A surface in three dimensions f (X, Y, Z) is define with:
- Any plane Ax+By+Cz+D=0, with constants a,b,c, and d,
- A sphere centered at the origin with a radius r: $x^2 + y^2 + z^2 - r^2 = 0$,
- Implicit functional form is define with: f(x,y) = 0 .

3. EXPLICIT CURVES

Explicit curves are single value. Mathematical function is y = f(x) can be plotted as curve. This function is explicit representation of curve. The main characteristics of explicit curves are:

- Do not allow for multiple values for a given argument,
- Cannot describe vertical tangents, as infinite slopes are hard to represent,
- Cannot represent all curves (vertical lines, circles).

4. PARAMETRIC CURVES

Curves have a parametric form called parametric curves. A curve in the plane is said to be parameterized if the set of coordinates on the curves (x,y,z) is represented as a function of a variable t. The variable t is called a parameter and the relations between x,y,z, and t are called a parametric equation:

$$x = f(t) \qquad y = g(t)$$

Each value of t defines a point (x,y)=(f(t),g(t)) that we can plot. The collection of points that we get by letting t be all possible values is the graph of the parametric equations and is called the parametric curve.

The parametric form of a curve is a function that assigns a position to values of the free parameters. That the parametric function is a vector-valued function. This example is a 2D curve, so the output of the function is a 2-D vector, in 3D it would be a 3 vector. It is simple and flexible. The parametric form is suitable for representing closed and multivalued curves. In parametric curves, each coordinate of a point on a curve is represented as a function of a single parameter. There are many curves that we cannot write down as a single equation in terms of x and y. The position vector of a point on the curve is fixed by the value of the parameter. Since a point on a parametric curve is specified by a single value of the parameter, the parametric form is axis-dependent.

5. BEZIER CURVES

These curves are discovered by the French engineer Pierre Bezier. A Bezier curve is particularly a kind of spline generated from a set of control points by forming a set of polynomial functions. These functions are computed from the coordinates of the control points. These curves can be generated under the control of other points. Tangents by using control points are used to generate curves.

Different types of curves are Simple, Quadratic, and Cubic.

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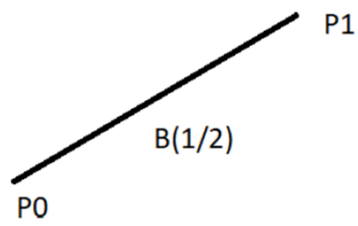
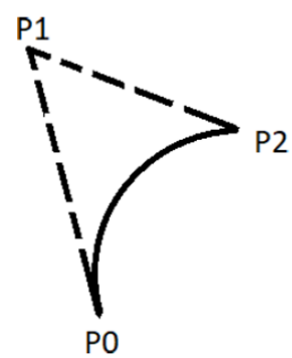
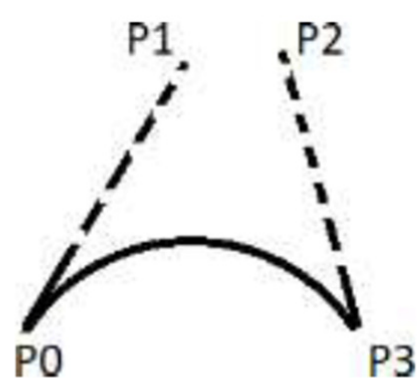
Simple Bezier curve is a straight line from the point.



Quadratic Bezier curve is determined by three control points.



The cubic Bezier curve is determined by four control points.



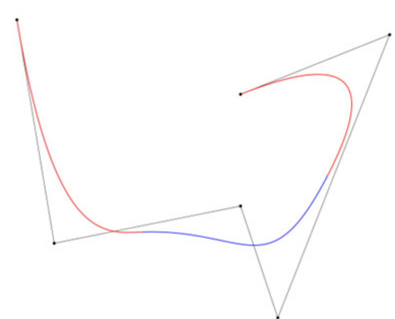
The main characteristics of Bezier curves are: Bezier curves are widely available and used in various CAD systems, in general graphics packages such as GL:

- The slope at beginning of the curve is along the line joining the first two control points and the slope at the end of the curve is along the line joining the last two points
- Bezier curve always passes through the first and last points i.e. p(o)=po, p(1, =pnlie)
- The curves lie entirely within the convex hall formed by the four control points
- The slope at the beginning of the curve is along the line joining the first two control points and the slope at the end of the curve is along the line joining the last two points.

6. B-SPLINE CURVES

The Bezier-curve produced by the Bernstein basis function has limited flexibility. First, the number of specified polygon vertices fixes the order of the resulting polynomial which defines the curve. The second limiting characteristic is that the value of the blending function is nonzero for all parameter values over the entire curve. The main characteristics of B-spline curves are:

- The maximum order of the curve is equal to the number of vertices of defining polygon.
- The degree of B-spline polynomial is independent on the number of vertices of defining polygon.
- B-spline allows the local control over the curve surface because each vertex affects the shape of a curve only over a range of parameter values where its associated basis function is nonzero and the curve exhibits the variation diminishing property.
- The curve generally follows the shape of defining polygon and any affine transformation can be applied to the curve by applying it to the vertices of defining polygon.
- The curve line within the convex hull of its defining polygon.



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